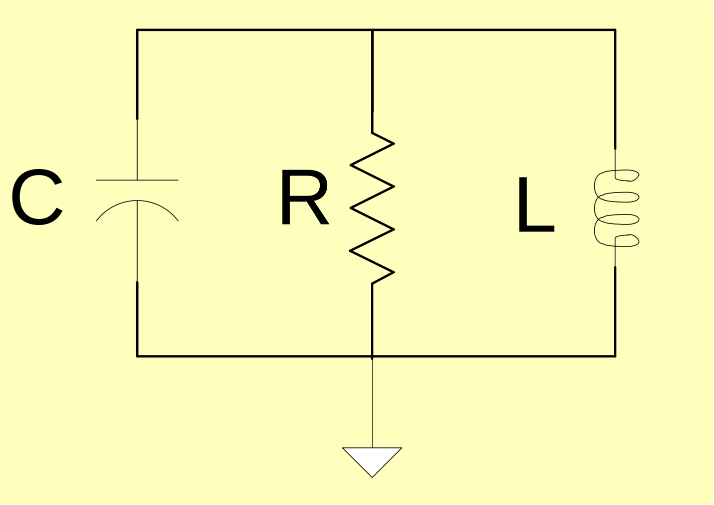
EE 230
Lecture 24

Waveform Generators

- Sinusoidal Oscillators

Quiz 18

Determine the characteristic equation for the following network without adding an excitation.



And the number is ?

1 3 8

5 4

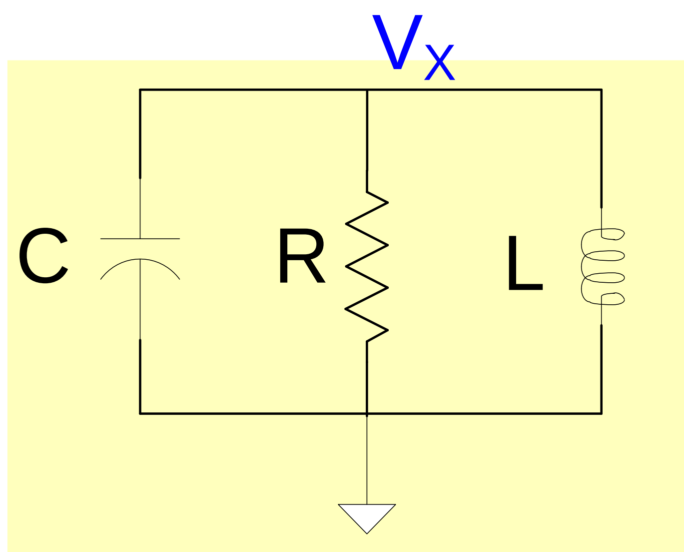
?

2 6

9 7

Quiz 18

Determine the characteristic equation for the following network without adding an excitation.



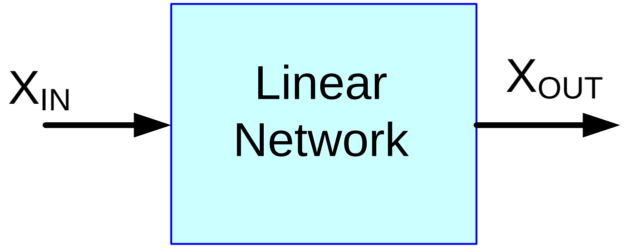
Solution:

$$V_X\left(G+sC+\frac{1}{sL}\right)=0$$

$$V_X\left(s^2+s\frac{1}{CR}+\frac{1}{LC}\right)=0$$

$$D\ s\ =s^2+s\frac{1}{CR}+\frac{1}{LC}$$

Review from Last Lecture



$$T(s)=\frac{N(s)}{D(s)}$$

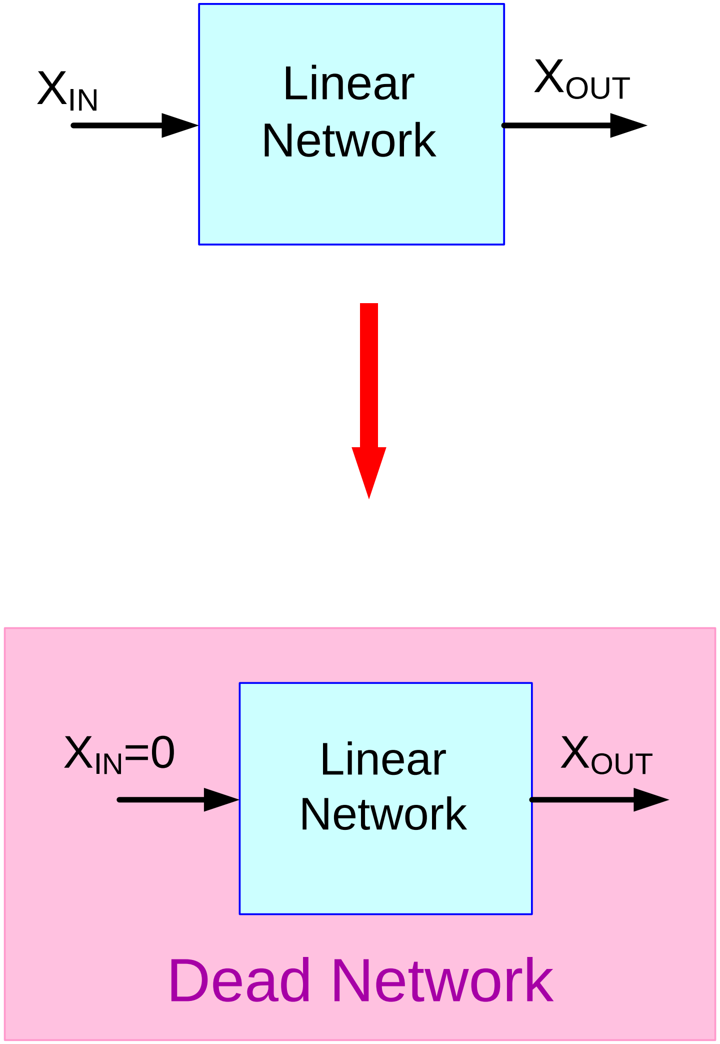
Theorem: The poles of any transfer function of a linear system are independent of where the excitation is applied and where the response is taken provided the “dead network” for the systems are the same.

Or, equivalently,

Theorem: The characteristic equation D(s) of a linear system are independent of where the excitation is applied and where the response is taken provided the “dead network” for the systems are the same.

Review from Last Lecture

Poles of a Network



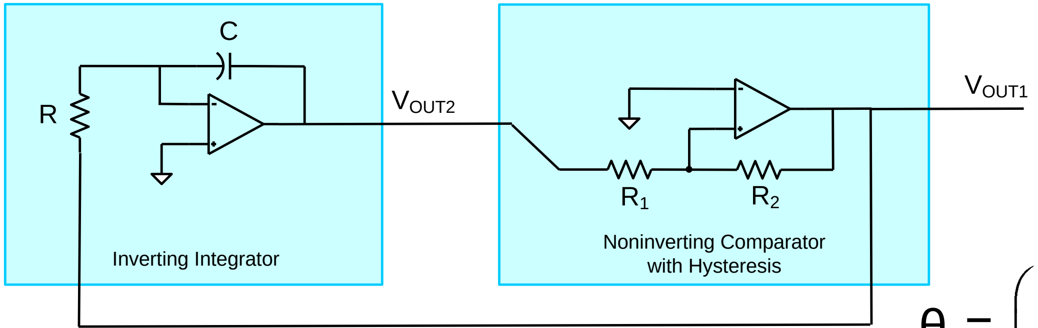
$$T\ s = \frac{N\ s}{D\ s}$$

$D\ s$

Theorem: The characteristic polynomial D(s) of a system can be obtained by assigning an output variable to the “dead network” of the system and using circuit analysis techniques to obtain an expression that involves only the output variable expressed as $X_0F(s)=0$ where F(s) is a polynomial. When expressed in this form, the characteristic polynomial of the system is $D(s)=F(s)$

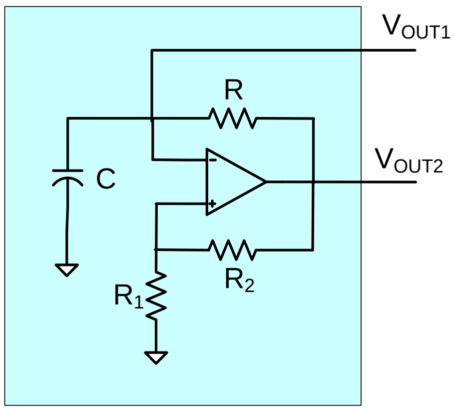
Review from Last Time:

Pole Locations of Waveform Generators

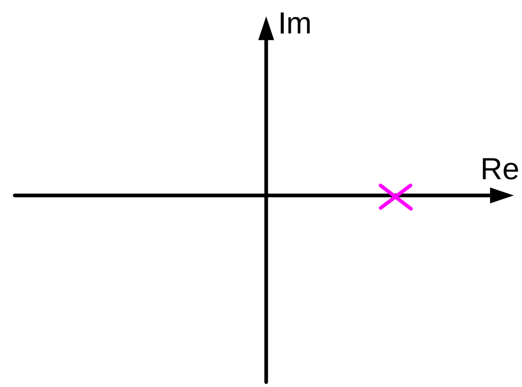


$$p=\left(\frac{1-\theta}{\theta}\right)\frac{1}{RC}$$

$$\theta=\left(\frac{R_1}{R_1+R_2}\right)$$



$$p=\left(\frac{R_2}{R_1}\right)\frac{1}{RC}$$



Both have a single pole on the positive real axis

Sinusoidal Oscillators

- The previous two circuits provided square wave and triangular (“triangularish”) outputs
  - previous two circuits had a RHP pole on positive real axis

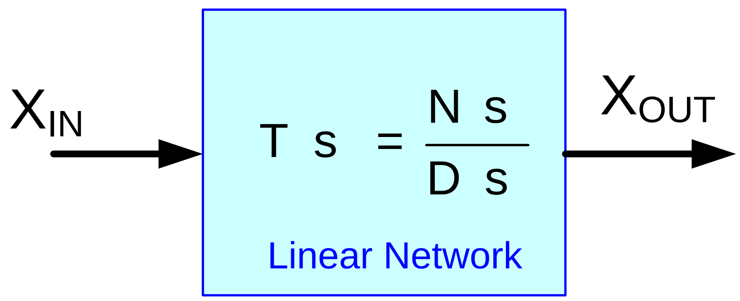
What properties of a circuit are needed to provide a sinusoidal output

- What circuits have these properties

What properties of a circuit are needed to provide a sinusoidal output?

- Insight into how a sinusoidal oscillator works
- Characteristic Equation Requirements for Sinusoidal Oscillation (Sec 13.1)
- Barkhausen Criterion (Sec 13.1)

Insight into how a sinusoidal oscillator works



$\{p_1, p_2, \ldots p_n\}$

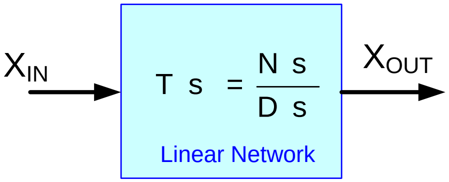
If e(t) is any excitation, no matter how small, can be expressed as

$$E(s) = \frac{N_E(s)}{D_E(s)} = \frac{N_E(s)}{(s-x_1)(s-x_2)\ldots(s-x_m)}$$

Then the output can be expressed, in the s-domain, as

$$R(s) = E(s)T(s) = \frac{N_E(s)}{D_E(s)} \bullet \frac{N(s)}{D(s)}$$

Insight into how a sinusoidal oscillator works



$\{p_1, p_2, \ldots\ p_n\}$

$$R(s)=\frac{N_E(s)}{D_E(s)}\bullet\frac{N(s)}{D(s)}$$

Using a partial fraction expansion for R(s) obtain

$$R(s)=\frac{a_1}{s-p_1}+\frac{a_2}{s-p_2}+\ldots+\frac{a_n}{s-p_n}+\left\{\frac{b_1}{s-x_1}+\frac{b_2}{s-x_2}+\ldots+\frac{b_m}{s-x_m}\right\}$$

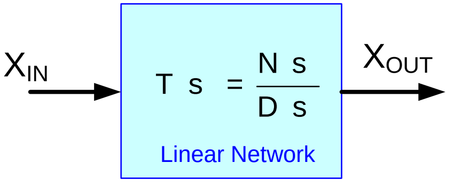
$$r(t)=\mathcal{L}^{-1}\ R(s)$$

Then the output can be expressed, in the time domain, as

$$r(t)=a_1e^{p_1t}+a_2e^{p_2t}+\ldots+a_ne^{p_nt}+\ b_1e^{x_1t}+b_2e^{x_2t}+\ldots+b_me^{x_mt}$$

If the excitation is very small and vanishes , or is zero, the term due to the excitation will vanish

Insight into how a sinusoidal oscillator works



$\{p_1, p_2, \ldots\, p_n\}$

$$R\;s = \frac{N_E\;s}{D_E\;s} \bullet \frac{N\;s}{D\;s}$$

Under the assumption that the excitation vanishes,

$$r\;t \simeq a_1 e^{p_1 t} + a_2 e^{p_2 t} + \ldots + a_n e^{p_n t}$$

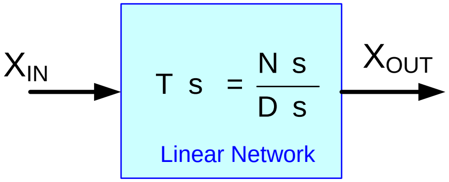
every pole can be expressed as the sum of a real part and imaginary part

$$p_k = \alpha_k + j\beta_k$$

(where the real part or the imaginary part may be 0)

$$r\;t \simeq a_1 e^{\alpha_1 t} e^{j\beta_1 t} + a_2 e^{\alpha_2 t} e^{j\beta_2 t} + \ldots + a_n e^{\alpha_n t} e^{j\beta_n t}$$

Insight into how a sinusoidal oscillator works

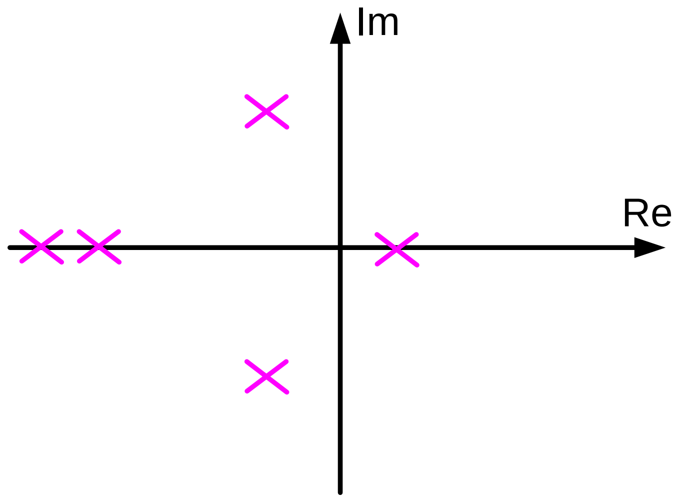


$\{p_1, p_2, \ldots\, p_n\}$

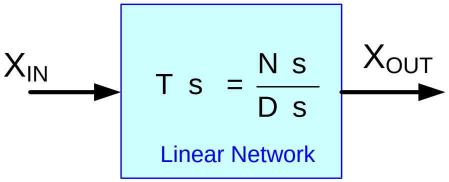
$$R\;\;s\;=\frac{N_E\;\;s}{D_E\;\;s}\bullet\frac{N\;\;s}{D\;\;s}$$

Theorem: If a linear network has a pole $p_k = \alpha_k + j\beta_k$ with a non-zero imaginary part, then its complex conjugate $p_k = \alpha_k - j\beta_k$ is also a pole of the network.

That is, all poles that are not on the real axis appear as complex-conjugate pairs



Insight into how a sinusoidal oscillator works



$\{p_1, p_2, \ldots\ p_n\}$

$$R\ s\ = \frac{N_E\ \ s}{D_E\ \ s} \bullet \frac{N\ \ s}{D\ s}$$

$$r\ \ t\ \simeq\ a_1 e^{\alpha_1 t} e^{j\beta_1 t}\ +\ a_2 e^{\alpha_2 t} e^{j\beta_2 t}\ + \ldots +\ a_n e^{\alpha_n t} e^{j\beta_n t}$$

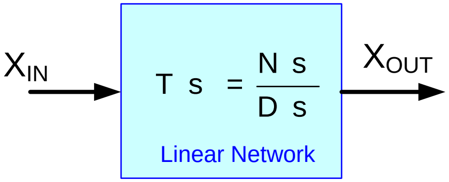
The terms on the right can be simply regrouped and renamed as

$$r\ \ t\ \simeq\ \hat{a}_1 e^{\hat{\alpha}_1 t} e^{j\hat{\beta}_1 t} + \hat{a}_2 e^{\alpha_r t} e^{-j\beta_1 t}\ +\ \hat{a}_3 e^{\hat{\alpha}_3 t} e^{j\hat{\beta}_3 t} + \hat{a}_4 e^{\alpha_3 t} e^{-j\beta_3 t}\ + \ldots +\ \hat{a}_k e^{\hat{\alpha}_k t} e^{j\hat{\beta}_k t} + \hat{a}_{k+1} e^{\alpha_k t} e^{-j\beta_k t}$$

$$+ \ldots +\ \hat{a}_{k+2} e^{\hat{\alpha}_{k+2} t}\ + \ldots\ \hat{a}_n e^{\hat{\alpha}_n t}$$

The first group of terms all correspond to complex conjugate pair poles and the second group to those that lie on the real axis

Insight into how a sinusoidal oscillator works



$\{p_1, p_2, \ldots\ p_n\}$

$$R\ \ s\ = \frac{N_E\ \ s}{D_E\ \ s} \bullet \frac{N\ \ s}{D\ \ s}$$

$$r\ \ t\ \simeq\ \hat{a}_1 e^{\hat{\alpha}_1 t} e^{j\hat{\beta}_1 t} + \hat{a}_2 e^{\hat{\alpha}_1 t} e^{-j\hat{\beta}_1 t}\ \ +\ \ \hat{a}_3 e^{\hat{\alpha}_3 t} e^{j\hat{\beta}_3 t} + \hat{a}_4 e^{\hat{\alpha}_3 t} e^{-j\hat{\beta}_3 t}\ \ + \ldots +\ \ \hat{a}_k e^{\hat{\alpha}_k t} e^{j\hat{\beta}_k t} + \hat{a}_{k+1} e^{\hat{\alpha}_k t} e^{-j\hat{\beta}_k t}$$

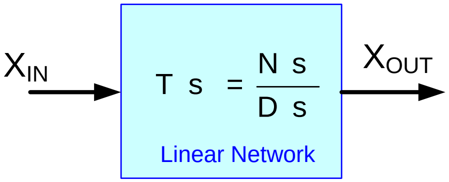
$$+ \ldots +\ \ \hat{a}_{k+2} e^{\hat{\alpha}_{k+2} t}\ \ + \ldots\ \hat{a}_n e^{\hat{\alpha}_n t}$$

Theorem: The coefficient in the partial fraction expansion corresponding to a pole with a non-zero imaginary part is the complex conjugate of the term corresponding to the complex conjugate pole.

$$r\ \ t\ \simeq\ \hat{a}_1 e^{\hat{\alpha}_1 t} e^{j\hat{\beta}_1 t} + \hat{a}_1^{'} e^{\hat{\alpha}_1 t} e^{-j\hat{\beta}_1 t}\ \ +\ \ \hat{a}_3 e^{\hat{\alpha}_3 t} e^{j\hat{\beta}_3 t} + \hat{a}_3^{'} e^{\hat{\alpha}_3 t} e^{-j\hat{\beta}_3 t}\ \ + \ldots +\ \ \hat{a}_k e^{\hat{\alpha}_k t} e^{j\hat{\beta}_k t} + \hat{a}_k^{'} e^{\hat{\alpha}_k t} e^{-j\hat{\beta}_k t}$$

$$+ \ldots +\ \ \hat{a}_{k+2} e^{\hat{\alpha}_{k+2} t}\ \ + \ldots\ \hat{a}_n e^{\hat{\alpha}_n t}$$

Insight into how a sinusoidal oscillator works



$$\{p_1,\ p_2,\ \ldots\ p_n\}$$

$$R\ \ s\ = \frac{N_E\ \ s}{D_E\ \ s} \bullet \frac{N\ \ s}{D\ \ s}$$

$$r\ \ t\ \simeq\ \hat{a}_1 e^{\hat{\alpha}_1 t} e^{j\hat{\beta}_1 t} + \hat{a}_1^{'} e^{\hat{\alpha}_1 t} e^{-j\hat{\beta}_1 t}\ \ +\ \ \hat{a}_3 e^{\hat{\alpha}_3 t} e^{j\hat{\beta}_3 t} + \hat{a}_3^{'} e^{\hat{\alpha}_3 t} e^{-j\hat{\beta}_3 t}\ +\ldots+\ \hat{a}_k e^{\hat{\alpha}_k t} e^{j\hat{\beta}_k t} + \hat{a}_k^{'} e^{\hat{\alpha}_k t} e^{-j\hat{\beta}_k t}$$

$$+\ldots+\ \ \hat{a}_{k+2} e^{\hat{\alpha}_{k+2} t}\ \ +\ldots\ \hat{a}_n e^{\hat{\alpha}_n t}$$

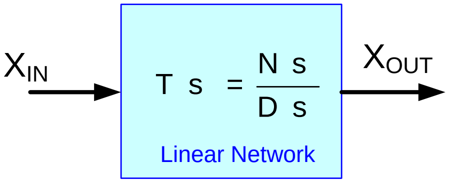
Consider now one of the complex conjugate pair terms $\hat{a}_k e^{\hat{\alpha}_k t} e^{j\hat{\beta}_k t} + \hat{a}_k^{'} e^{\alpha_k t} e^{-j\beta_k t}$

$$\hat{a}_k e^{\hat{\alpha}_k t} e^{j\hat{\beta}_k t} + \hat{a}_k^{'} e^{\hat{\alpha}_k t} e^{-j\hat{\beta}_k t}\ \ = 2\left|\hat{a}_k\right| e^{\hat{\alpha}_k t} \cos \hat{\beta}_k t$$

$$r\ \ t\ \simeq\ 2\left|\hat{a}_1\right| e^{\hat{\alpha}_1 t} \cos \hat{\beta}_1 t\ \ +\ \ 2\left|\hat{a}_3\right| e^{\hat{\alpha}_3 t} \cos \hat{\beta}_3 t\ +\ldots+\ 2\left|\hat{a}_k\right| e^{\hat{\alpha}_k t} \cos \hat{\beta}_k t$$

$$+\ldots+\ \ \hat{a}_{k+2} e^{\hat{\alpha}_{k+2} t}\ \ +\ldots\ \hat{a}_n e^{\hat{\alpha}_n t}$$

Insight into how a sinusoidal oscillator works



$\{p_1, p_2, \ldots p_n\}$

$$R(s) = \frac{N_E(s)}{D_E(s)} \bullet \frac{N(s)}{D(s)}$$

$$r(t) \simeq 2|\hat{a}_1|e^{\hat{\alpha}_1 t}\cos\hat{\beta}_1 t + 2|\hat{a}_3|e^{\hat{\alpha}_3 t}\cos\hat{\beta}_3 t + \ldots + 2|\hat{a}_k|e^{\hat{\alpha}_k t}\cos\hat{\beta}_k t$$

$$+ \ldots + \hat{a}_{k+2}e^{\hat{\alpha}_{k+2} t} + \ldots \hat{a}_n e^{\hat{\alpha}_n t}$$

Consider now three cases, the real part of the pole is negative, the real part of the pole is positive, and the real part of the pole is 0

It the real part of the pole is negative, the corresponding term in r(t) will vanish

It the real part of the pole is positive, the corresponding term in r(t) will diverge to +/-∞

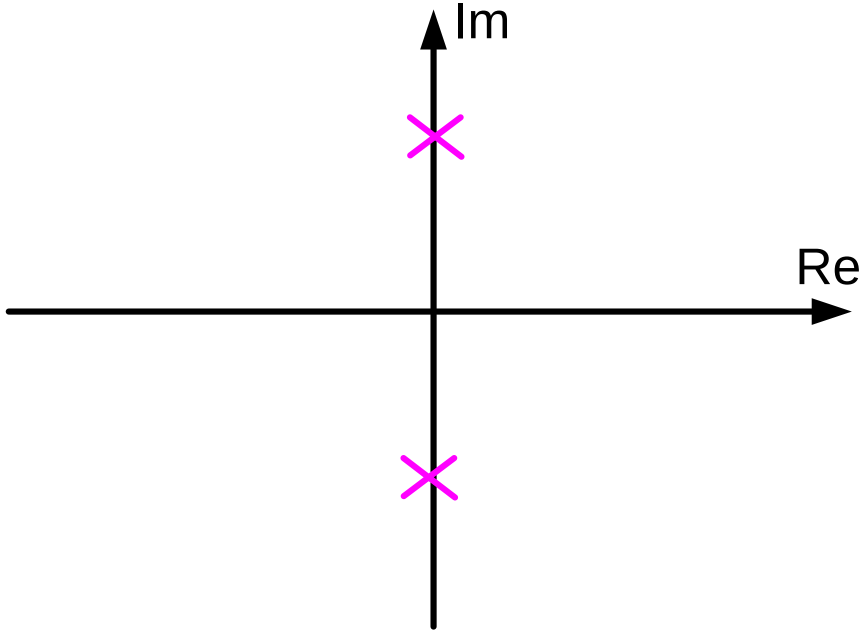
It the real part of the pole is zero, the corresponding term in r(t) will be a sinusoidal waveform that will persist forever

The condition for sinusoidal oscillation should be apparent !

What properties of a circuit are needed to provide a sinusoidal output?

- Insight into how a sinusoidal oscillator works
- Characteristic Equation Requirements for Sinusoidal Oscillation (Sec 13.1)
- Barkhausen Criterion (Sec 13.1)

Sinusoidal Oscillation



A circuit with a single complex conjugate pair of poles on the imaginary axis at +/- jβ will have a sinusoidal output given by

$$X_{OUT}\ \ t\ =2\left|\hat{a}_k\right|\sin(\beta t+\theta)$$

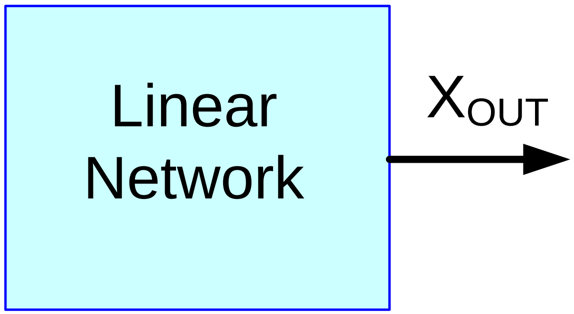
The frequency of oscillation will be β rad/sec but the amplitude and phase are indeterminate

Sinusoidal Oscillation Criteria

A network that has a single complex conjugate pair on the imaginary axis at $\pm j\omega$ and no RHP poles will have a sinusoidal output of the form $X_0(t) = A\sin(\omega t + \theta)$

A and θ can not be determined by properties of the linear network

Characteristic Equation Requirements for Sinusoidal Oscillation



Characteristic Equation Oscillation Criteria:

If the characteristic equation D(s) has exactly one pair of roots on the imaginary axis and no roots in the RHP, the network will have a sinusoidal signal on every nongrounded node.

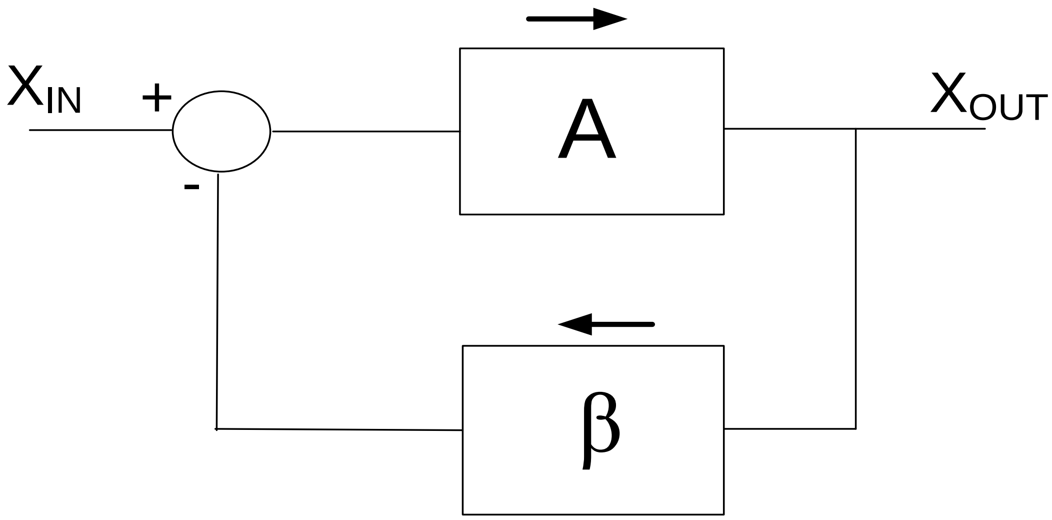
What properties of a circuit are needed to provide a sinusoidal output?

- Insight into how a sinusoidal oscillator works
- Characteristic Equation Requirements for Sinusoidal Oscillation (Sec 13.1)

→ Barkhausen Criterion (Sec 13.1)

Barkhausen Oscillation Criteria

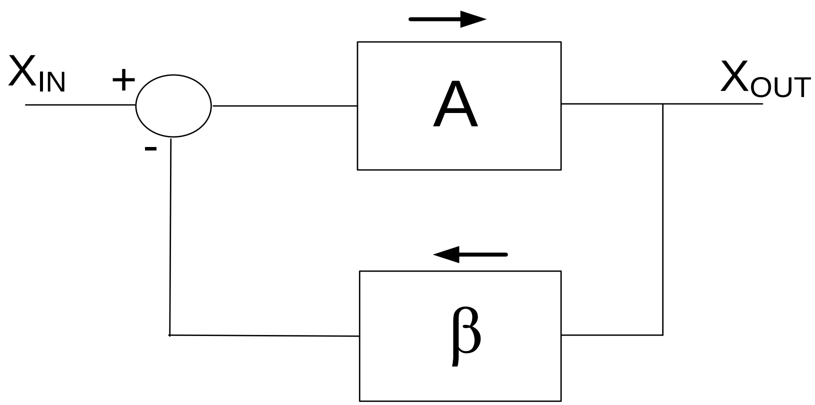
Consider a basic feedback amplifier



$$A_{FB} = \frac{A}{1 + A\beta}$$

Aβ is termed the loop gain

Barkhausen Oscillation Criteria



$$A_{FB} = \frac{A}{1 + A\beta}$$

Barkhausen Oscillation Criteria:

A feedback amplifier will have sustained oscillation if Aβ=-1

There are many subtly different ways various authors present Barkhausen criteria but all invariably state, in some way, that must have Aβ=-1

Some state it must occur at only one frequency, others make comments about waveshape, Sedra and Smith convey right idea but are neither rigorous or completely correct, and most other authors have a similar problem

Relationship between Barkhausen Criteria for Oscillation and Characteristic Equation Criteria

Barkhausen Oscillation Criteria

A feedback amplifier will have sustained oscillation if $A\beta=-1$

Characteristic Equation Oscillation Criteria

If the characteristic equation D(s) has exactly one pair of roots on the imaginary axis and no roots in the RHP, the network will have a sinusoidal signal on every nongrounded node

Differences:

1. Barkhausen requires a specific feedback amplifier architecture

2. Sustained oscillation says nothing about waveshape

Relationship between Barkhausen Criteria for Oscillation and Characteristic Equation Criteria

Barkhausen Oscillation Criteria

A feedback amplifier will have sustained oscillation if $A\beta=-1$

Characteristic Equation Oscillation Criteria (CEOC)

If the characteristic equation D(s) has exactly one pair of roots on the imaginary axis and no roots in the RHP, the network will have a sinusoidal signal on every nongrounded node

If a network can be represented as a basic feedback amplifier, then

$$A_{FB}(s)=\frac{A}{1+A\beta}=\frac{N(s)}{D(s)}$$

If D(s) criteria is satisfied, then at the poles $p=\pm j\omega$, $1+A\beta=0$ or equivalently $A\beta=-1$

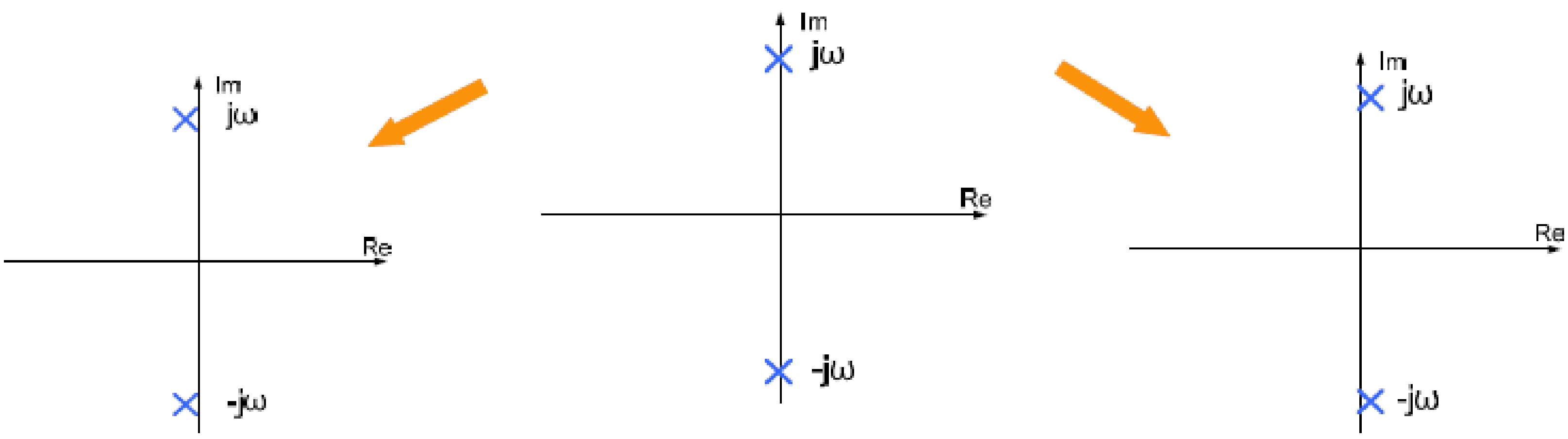
But, if a network has $A\beta=-1$, even at a single pole pair, there may be other poles in the RHP that would violate the CEOC needed for sinusoidal oscillation

If sinusoidal oscillation is required, be very careful about using Barkhausen Criteria

Characteristic Equation Oscillation Criteria (CEOC)

If the characteristic equation D(s) has exactly one pair of roots on the imaginary axis and no roots in the RHP, the network will have a sinusoidal signal on every nongrounded node

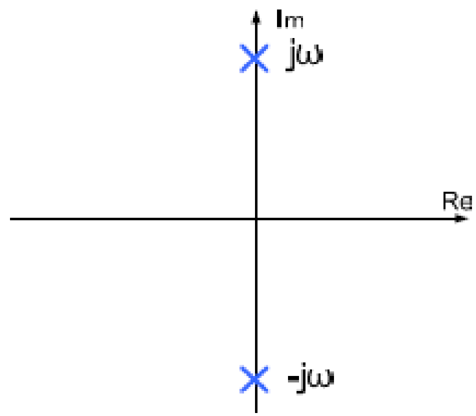
But – it is impossible to place a pair of poles of D(s) precisely on the imaginary axis !



Characteristic Equation Oscillation Criteria (CEOC)

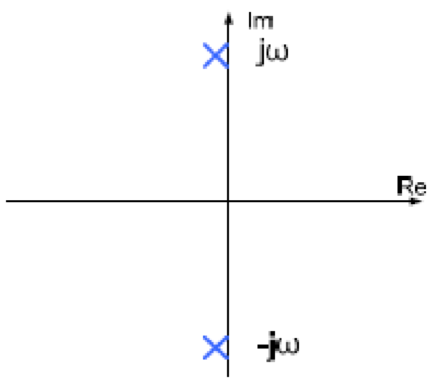
But – it is impossible to place a pair of poles of D(s) precisely on the imaginary axis !

If $p = \alpha \pm j\beta$



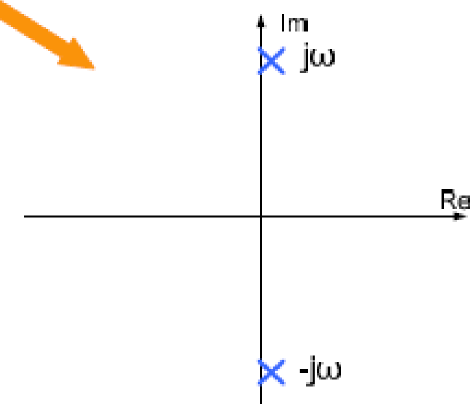
$X(t) = A\sin(\omega t + \theta)$

Impossible to achieve



$X(t) = e^{\alpha t} A\sin(\omega t + \theta)$

Sinusoidal signal will decay to 0



$X(t) = e^{\alpha t} A\sin(\omega t + \theta)$

Output will diverge to ∞

Characteristic Equation Oscillation Criteria (CEOC)

If the characteristic equation D(s) has exactly one pair of roots on the imaginary axis and no roots in the RHP, the network will have a sinusoidal signal on every nongrounded node.

Sinusoidal Oscillator Design Strategy

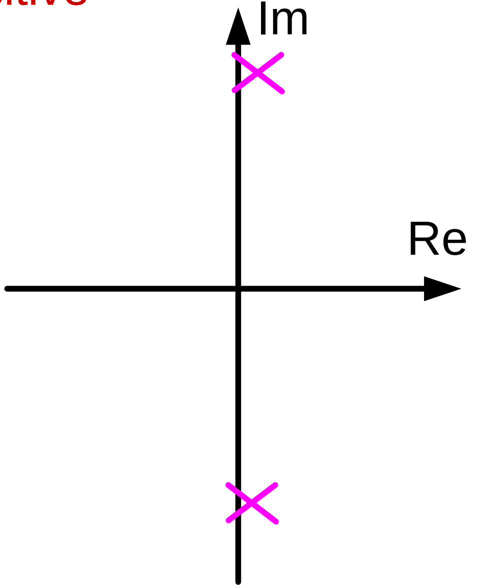
Build networks with exactly one pair of complex conjugate poles slightly in the RHP and use nonlinearities in the amplifier part of the network to limit the amplitude of the output. i.e. $p=\alpha \pm j\beta$ where α is very small but positive

- Nonlinearity in amplifier will result in a small amount of distortion
- Frequency of oscillation will deviate slightly from β
- Poles must be far enough in the RHP so process and temperature variations do not cause movement back into LHP because if that happened, oscillation would cease!

Sinusoidal Oscillator Design Strategy

Build networks with exactly one pair of complex conjugate poles slightly in the RHP and use nonlinearities in the amplifier part of the network to limit the amplitude of the output.

i.e. $p=\alpha \pm j\beta$ where α is very small but positive



Know Barkhausen Crieteria to answer interviewers questions but use CEOC criteria to design sinusoidal oscillators !

Sinusoidal Oscillators

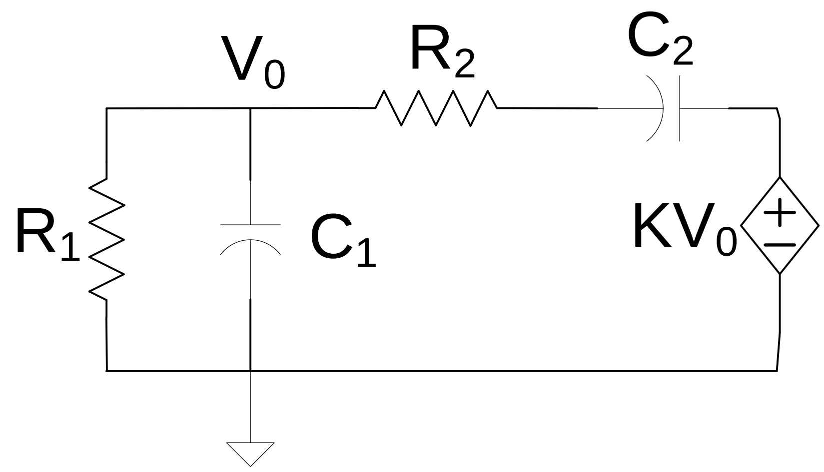
- The previous two circuits provided square wave and triangular (“triangularish”) outputs
  - previous two circuits had a RHP pole on positive real axis

- What properties of a circuit are needed to provide a sinusoidal output

→ What circuits have these properties

Sinusoidal Oscillator Design

Consider the following circuit:



By KCL

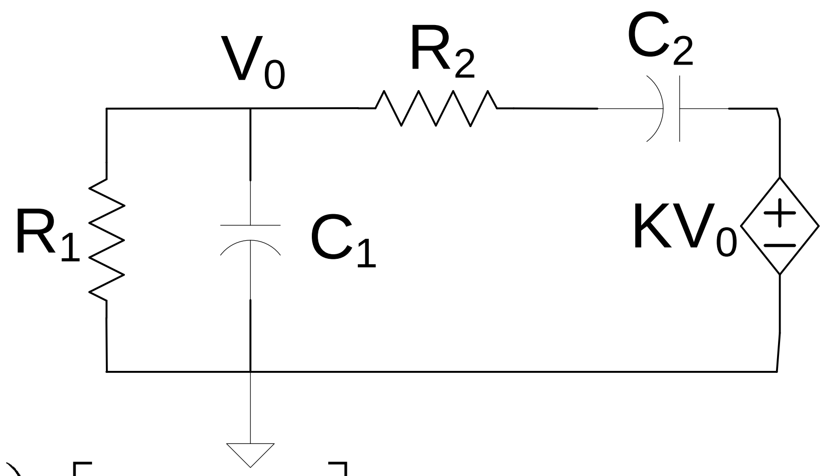
$$V_0\left(G_1+sC_1+\left[\frac{sC_2G_2}{G_2+sC_2}\right]\right)=KV_0\left[\frac{sC_2G_2}{G_2+sC_2}\right]$$

$$V_0\left(s^2+s\left(\frac{1}{R_2C_2}+\frac{1}{R_1C_1}+\frac{1-K}{R_2C_1}\right)+\left[\frac{1}{R_1R_2C_1C_2}\right]\right)=0$$

$$D\ \ s\ \ =s^2+s\left(\frac{1}{R_2C_2}+\frac{1}{R_1C_1}+\frac{1-K}{R_2C_1}\right)+\left[\frac{1}{R_1R_2C_1C_2}\right]$$

Sinusoidal Oscillator Design

Consider the following circuit:



$$D\quad s\ = s^2+s\left(\frac{1}{R_2C_2}+\frac{1}{R_1C_1}+\frac{1-K}{R_2C_1}\right)+\left[\frac{1}{R_1R_2C_1C_2}\right]$$

If the coefficient of the s term is set to 0, will have cc poles at

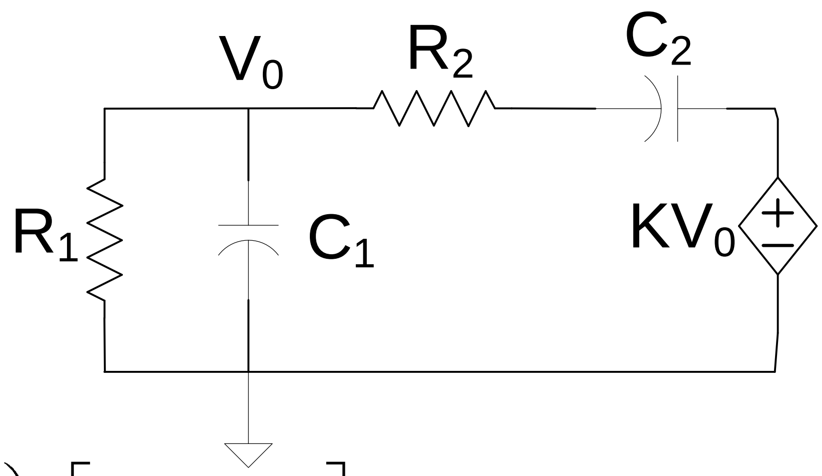
$$s = \pm j\sqrt{\frac{1}{R_1R_2C_1C_2}}$$

Thus, it the coefficient of the s term vanishes, it will be a sinusoidal oscillator of frequency

$$\omega_{OSC} = \sqrt{\frac{1}{R_1R_2C_1C_2}}$$

Sinusoidal Oscillator Design

Consider the following circuit:



$$D(s) = s^2 + s\left(\frac{1}{R_2C_2} + \frac{1}{R_1C_1} + \frac{1-K}{R_2C_1}\right) + \left[\frac{1}{R_1R_2C_1C_2}\right] \qquad \omega_{OSC} = \sqrt{\frac{1}{R_1R_2C_1C_2}}$$

Oscillation Condition:

$$\frac{1}{R_2C_2} + \frac{1}{R_1C_1} + \frac{1-K}{R_2C_1} = 0$$

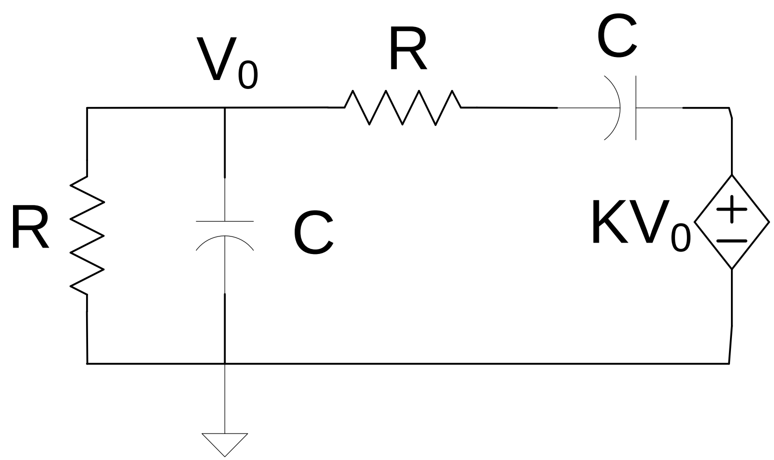
This is achieved by having K satisfy the equation

$$K = 1 + \frac{C_1}{C_2} + \frac{R_2}{R_1}$$

Sinusoidal Oscillator Design

Consider the special practical case where
R_1=R_2=R and C_1=C_2=C:

$$K = 1 + \frac{C_1}{C_2} + \frac{R_2}{R_1} = 3$$



$$\omega_{OSC} = \frac{1}{R\,C}$$

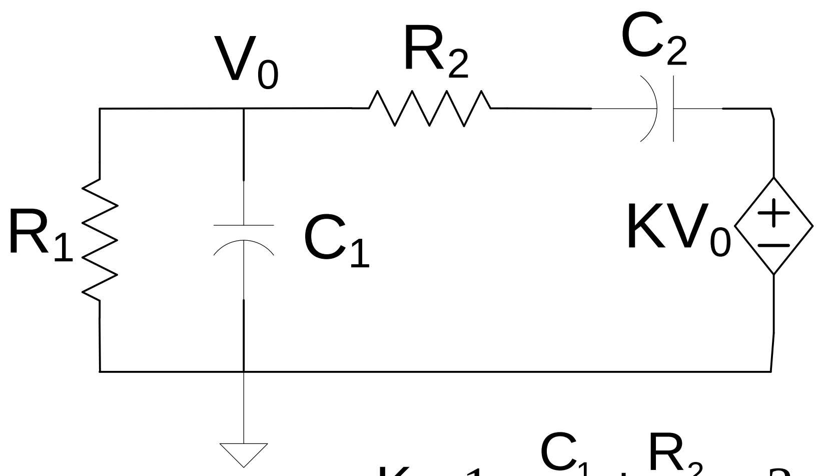
This is termed the Wein Bridge Oscillator

One of the most popular sinusoidal oscillator structures

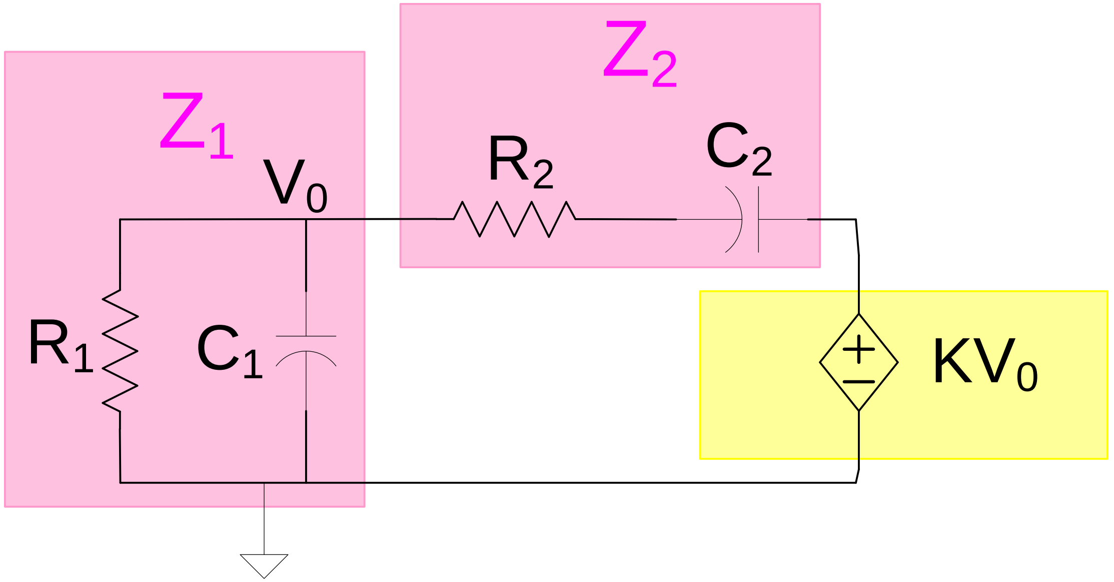
Practically make K slightly larger than 3 and judiciously manage the nonlinearities to obtain low distortion

The Wein-Bridge Oscillator

Another Perspective:

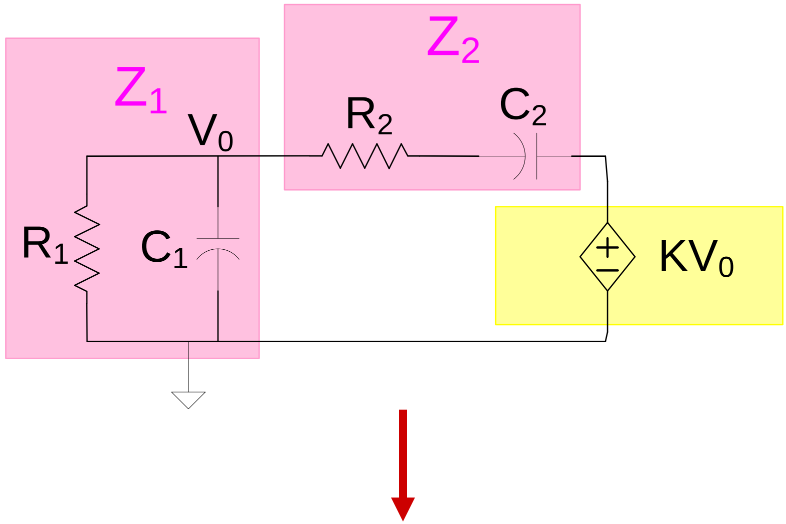


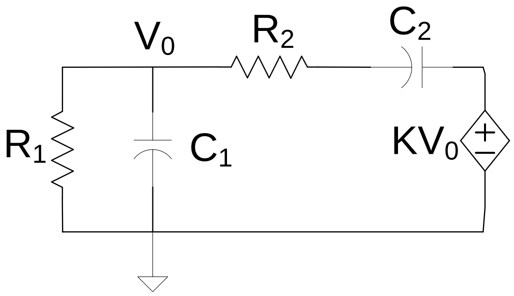
$$K = 1 + \frac{C_1}{C_2} + \frac{R_2}{R_1} = 3$$



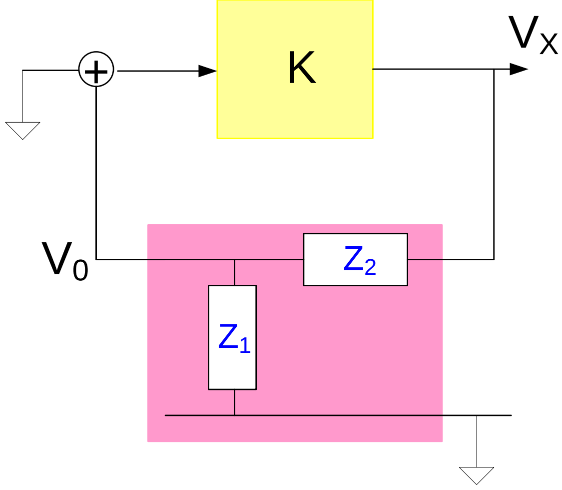
The Wein-Bridge Oscillator

Another Perspective:





$$K = 1 + \frac{C_1}{C_2} + \frac{R_2}{R_1} = 3$$



Note this is a feedback amplifier with gain K and

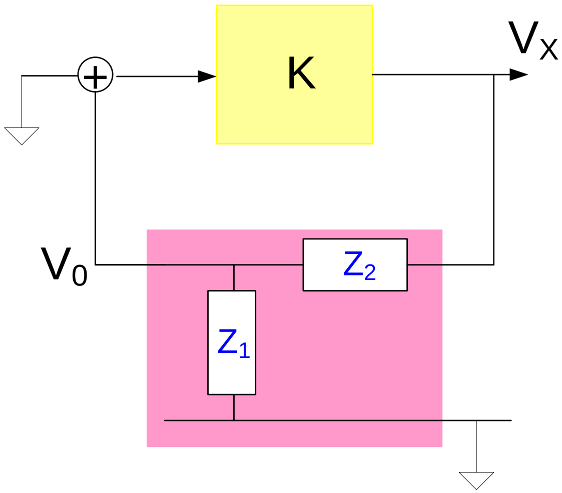
$$\beta = \frac{Z_1}{Z_1 + Z_2}$$

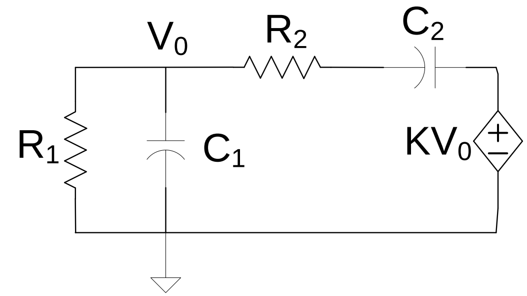
Lets check Barkhausen Criteria for this circuit

End of Lecture 24

The Wein-Bridge Oscillator

Lets check Barkhausen Criteria for this circuit





$$\beta = \frac{Z_1}{Z_1+Z_2}$$

$$K = 1 + \frac{C_1}{C_2} + \frac{R_2}{R_1} = 3$$

$$K\beta = \frac{Z_1}{Z_1+Z_2} = \frac{KR_1C_1s}{s^2R_1R_2C_1C_2+s\left(R_1C_2+R_1C_1+R_2C_2\right)+1}$$

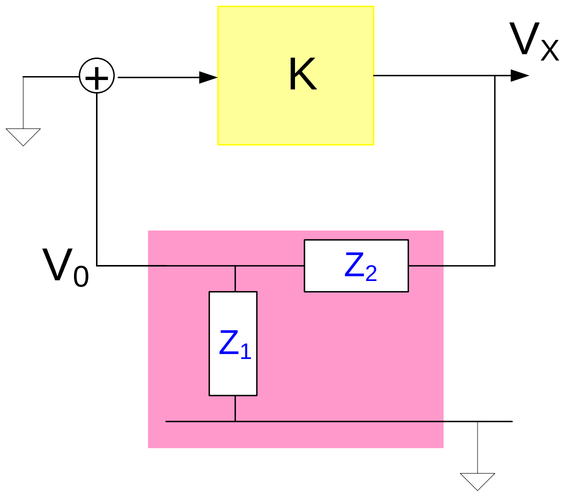
Setting Kβ= -1, obtain

$$\frac{KR_1C_1s}{s^2R_1R_2C_1C_2+s\left(R_1C_2+R_1C_1+R_2C_2\right)+1} = -1$$

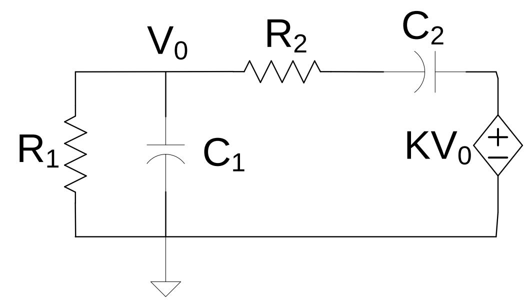
$$s^2R_1R_2C_1C_2+s\left(R_1C_2+R_1C_1+R_2C_2-KR_1C_1\right)+1=0$$

The Wein-Bridge Oscillator

Lets check Barkhausen Criteria for this circuit



$$\beta = \frac{Z_1}{Z_1+Z_2}$$



$$K = 1 + \frac{C_1}{C_2} + \frac{R_2}{R_1} = 3$$

Putting in s= jω, obtain the Barkhausen criteria

$$\left[1-\omega^2 R_1R_2C_1C_2\right]+j\left[\omega\ R_1C_2+R_1C_1+R_2C_2-KR_1C_1\right]=0$$

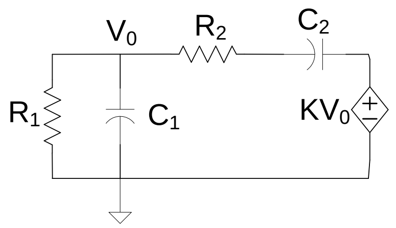
Solving, must have (from the imaginary part)

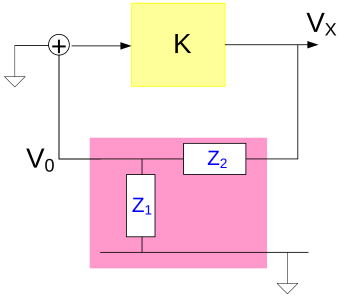
$$K = 1 + \frac{C_1}{C_2} + \frac{R_2}{R_1}$$

And this will occur at the oscillation frequency of (from real part)

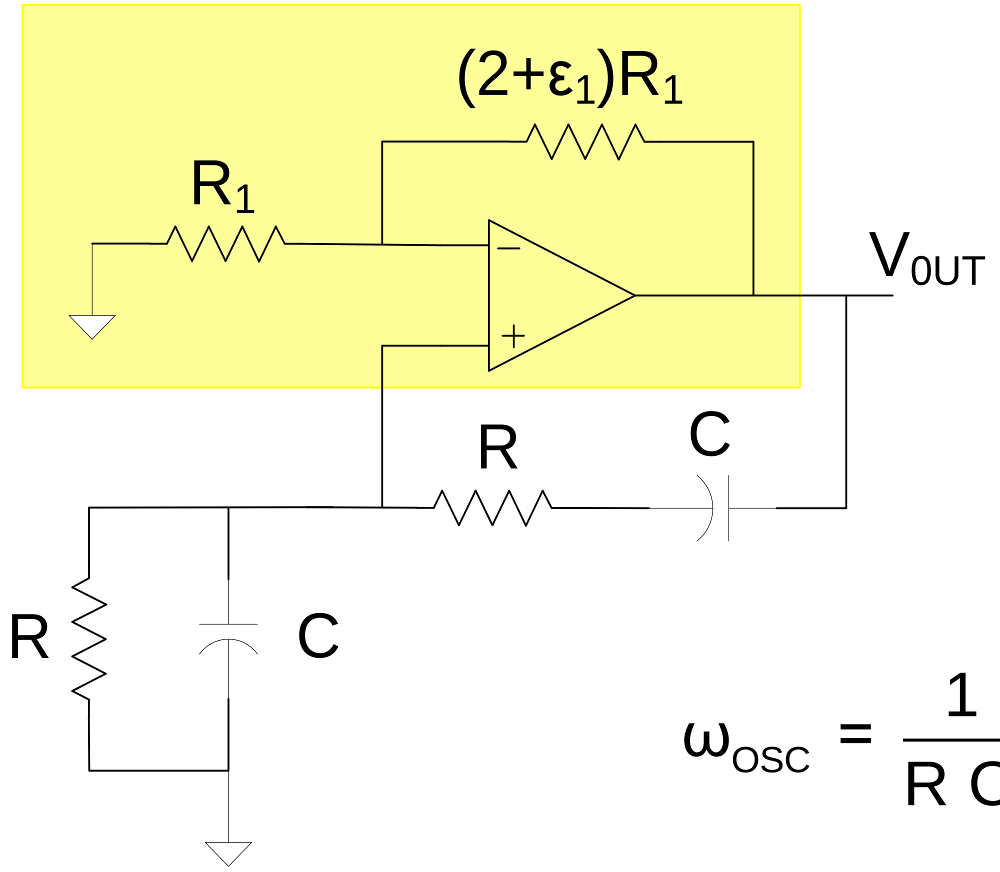
$$\omega_{OSC} = \sqrt{\frac{1}{R_1R_2C_1C_2}}$$

The Wein-Bridge Oscillator





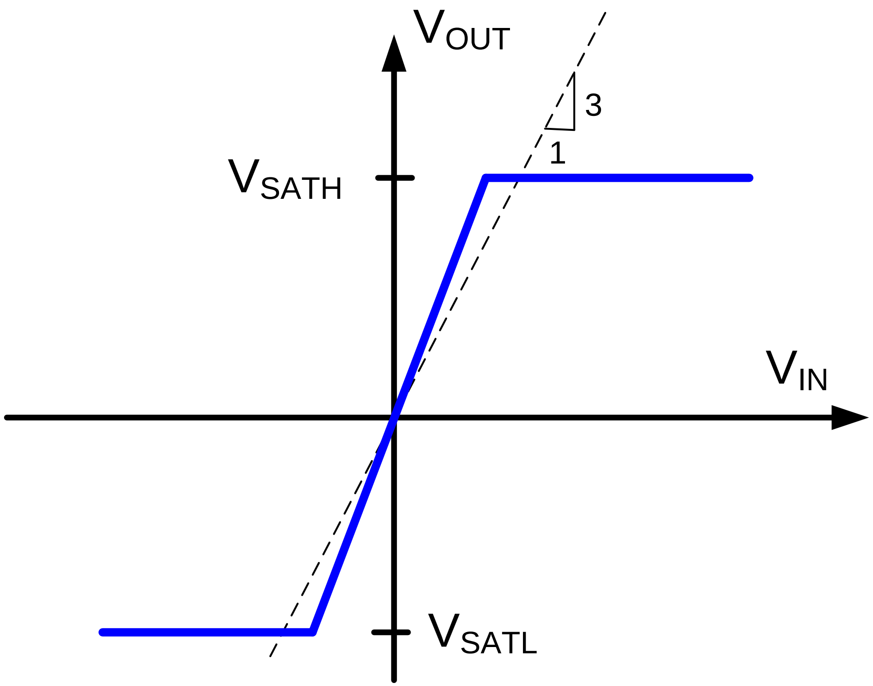
Basic implementation for the equal R, equal C case

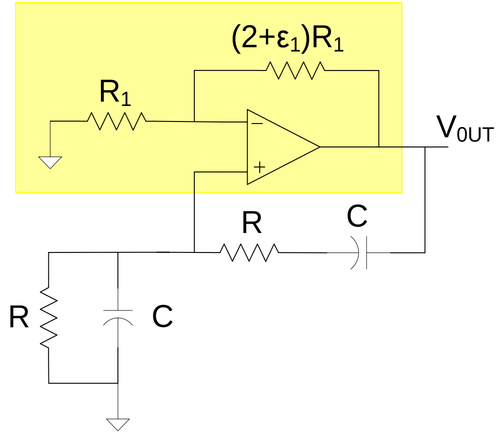


$$\omega_{OSC} = \frac{1}{R\,C}$$

The Wein-Bridge Oscillator

Amplifier Transfer Characteristics

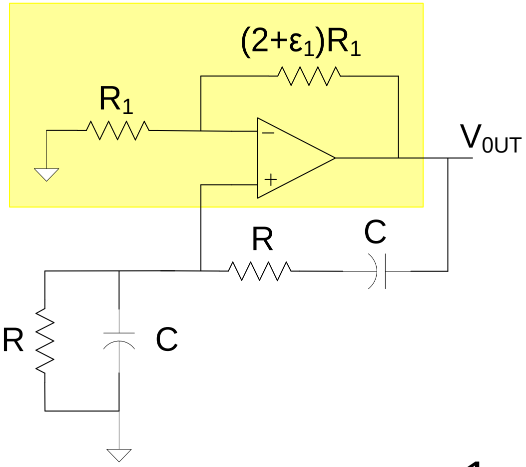




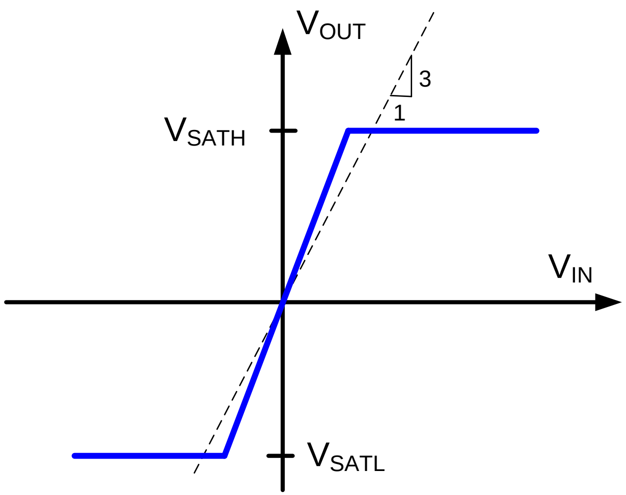
$$\omega_{OSC} = \frac{1}{R\,C}$$

Slope slightly larger than 3
Amplitude of oscillation will be approximately V_{SATH} (assuming V_{SATH}=-V_{STATL})
Distortion introduced by the abrupt nonlinearities when clipping occurs

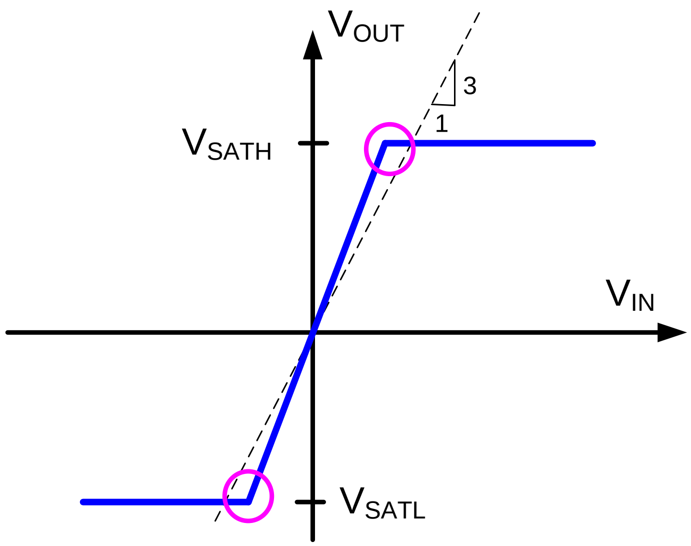
The Wein-Bridge Oscillator



Amplifier Transfer Characteristics



$$\omega_{OSC} = \frac{1}{R\,C}$$



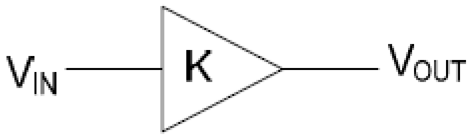
Abrupt nonlinearities cause distortion

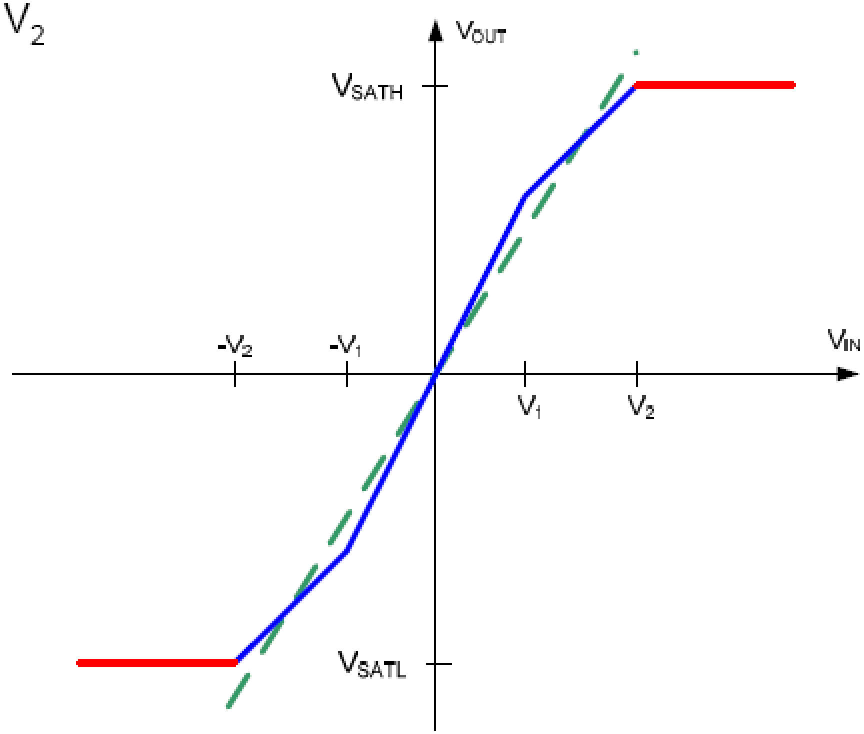
Better performance (reduced nonlinearity) can be obtained by introducing less abrupt nonlinearities to limit amplitude

Can we do this?

Obtain slope >3 for $-V_1 < V_{IN} < V_1$ and slope <3 for $V_1 < V_{IN} < V_2$ and for $-V_2 < V_{IN} < -V_1$

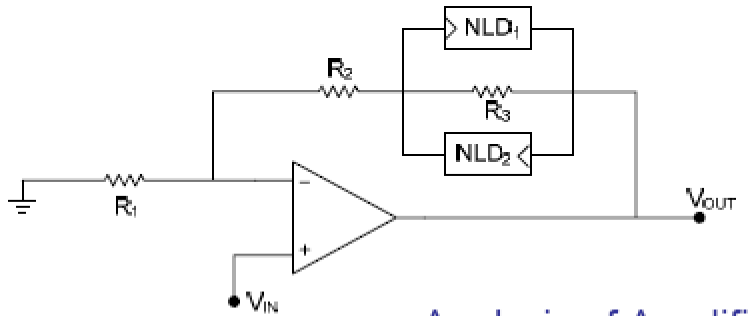
Limit V_{IN} to interval $-V_2 < V_{IN} < V_2$

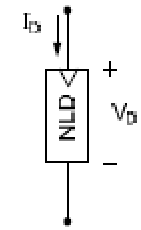




- If possible, hard nonlinearity associated with amplifier saturation will not be excited
- Dramatic reduction in distortion is anticipated

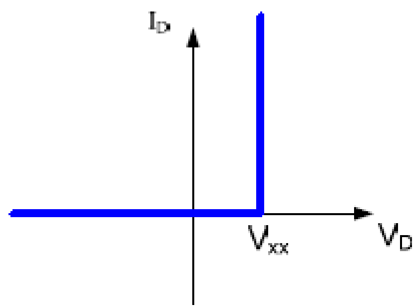
Consider:

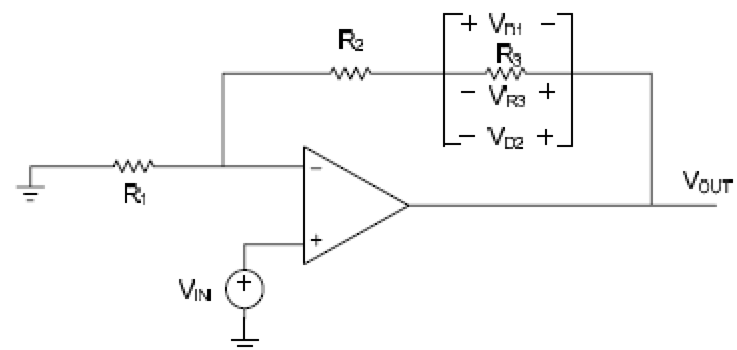




Analysis of Amplifier Circuit:

Case 1: $I_{D1} = 0$ and $I_{D2} = 0$





NLD Transfer Characteristics

$V_D = V_{XX}$ for $I_D > 0$

$I_D = 0$ for $V_D < V_{XX}$

(will assume $V_{XX} > 0$)

Solution:

$$V_{OUT} = \left[1 + \frac{(R_2 + R_3)}{R_1}\right] V_{IN}$$

Must determine where this part of the solution is valid

Valid for $V_{D1} < V_{XX}$ and $V_{D2} < V_{XX}$

but $V_{D1} = -V_{R3}$ and $V_{D2} = V_{R3}$

thus, valid for

$V_{R3} > -V_{XX}$ and $V_{R3} < V_{XX}$

but
$$V_{R3} = \frac{R_3}{R_2 + R_3}(V_o - V_{in})$$

$$V_{R3} = \frac{R_3}{R_2 + R_3}\left[1 + \frac{R_2 + R_3}{R_1} - 1\right]V_{in}$$

$$V_{R3} = \frac{R_3}{R_2 + R_3}\left[\frac{R_2 + R_3}{R_1}\right]V_{in}$$

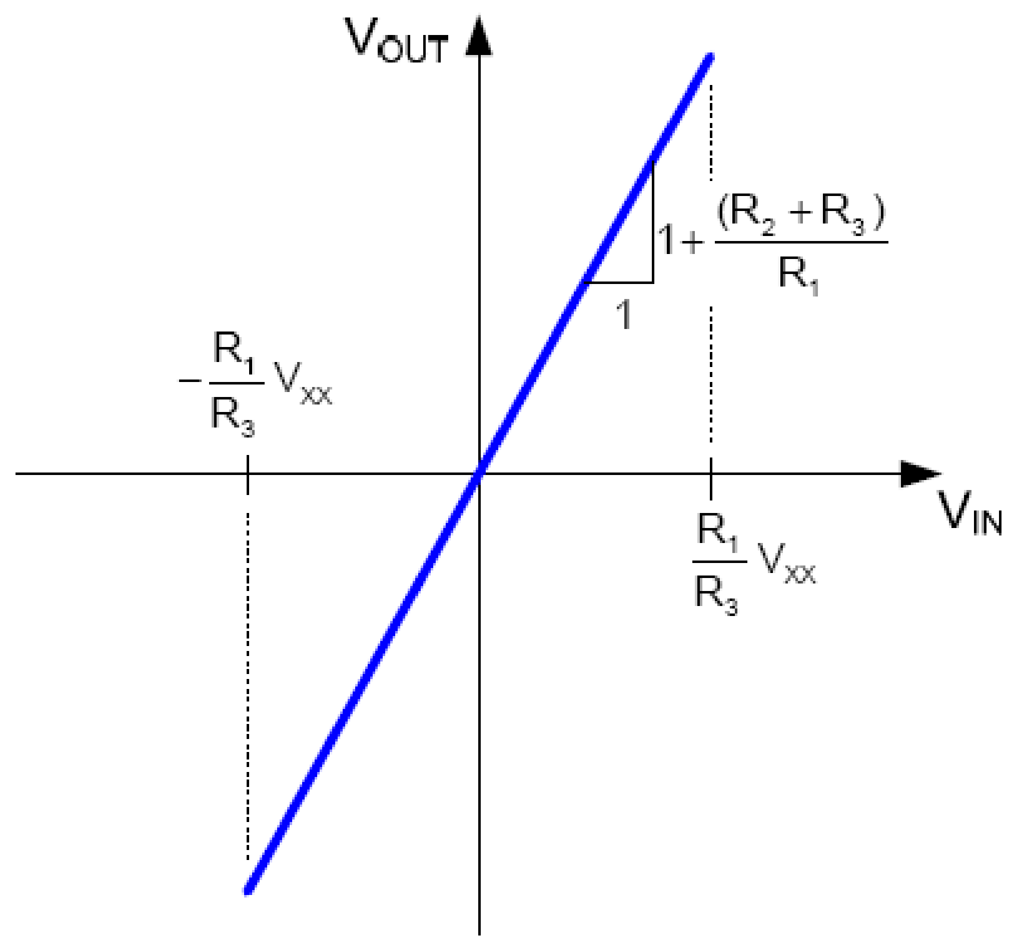
$$V_{R3} = \frac{R_3}{R_1}V_{in}$$

$\therefore$ valid for

$$\frac{R_3}{R_1}V_{IN} < V_{XX} \quad \text{and} \quad \frac{R_3}{R_1}V_{IN} > -V_{XX}$$

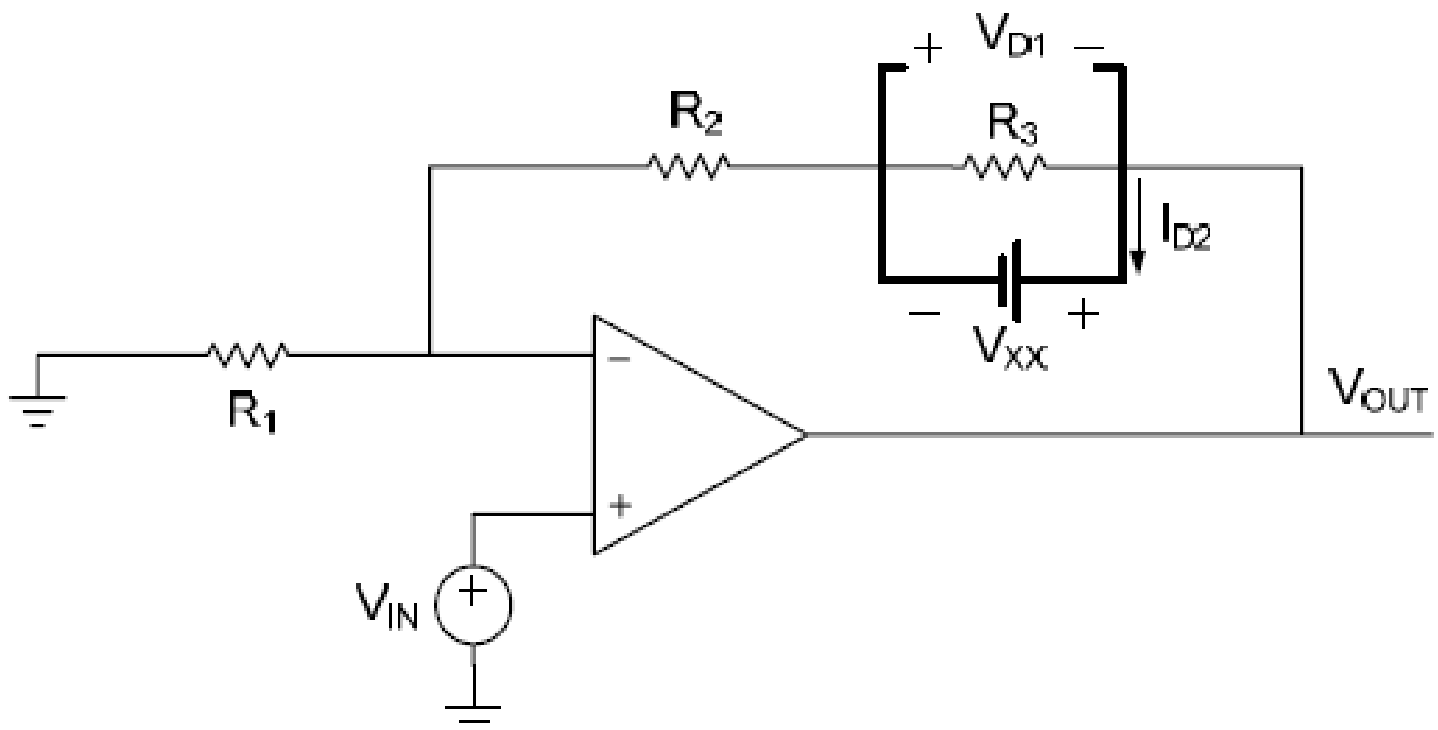
$$-\frac{R_1}{R_3}V_{XX} < V_{IN} < \frac{R_1}{R_3}V_{XX}$$

Graph of solution for Case 1



Case 2: NLD$_2$ is in the conducting state ($V_{D2} = V_{XX}$)

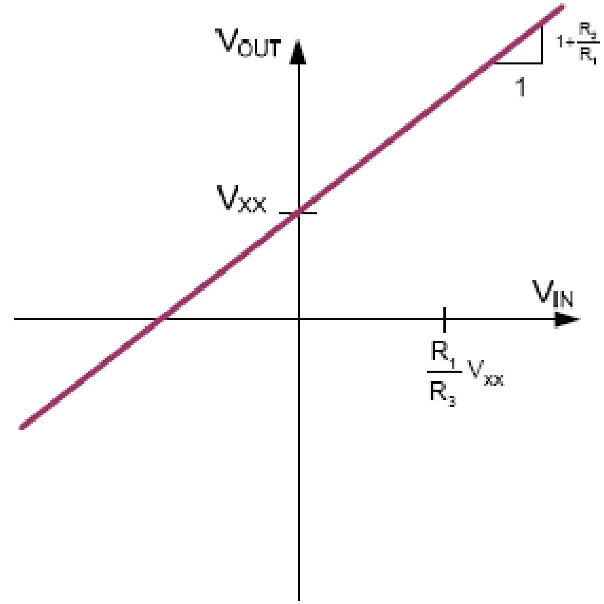
NLD$_1$ is in the nonconducting state ($I_{D1} = 0$)



Solution for Case 2 Continued

Applying superposition we obtain

$$V_{OUT} = V_{IN}\left(1+\frac{R_2}{R_1}\right)+V_{xx}$$



This solution is valid for $V_{D1} < 0$ and $I_{D2} > 0$

But: $V_{D1} = -V_{XX}$ and $I_{D2} = \dfrac{V_{OUT}-V_{IN}-V_{XX}}{R_2} - \dfrac{V_{XX}}{R_3}$

Substituting the validity conditions, we obtain

$$-V_{XX} < 0 \quad \text{and} \quad \frac{V_{OUT}-V_{IN}-V_{XX}}{R_2} - \frac{V_{XX}}{R_3} > 0$$

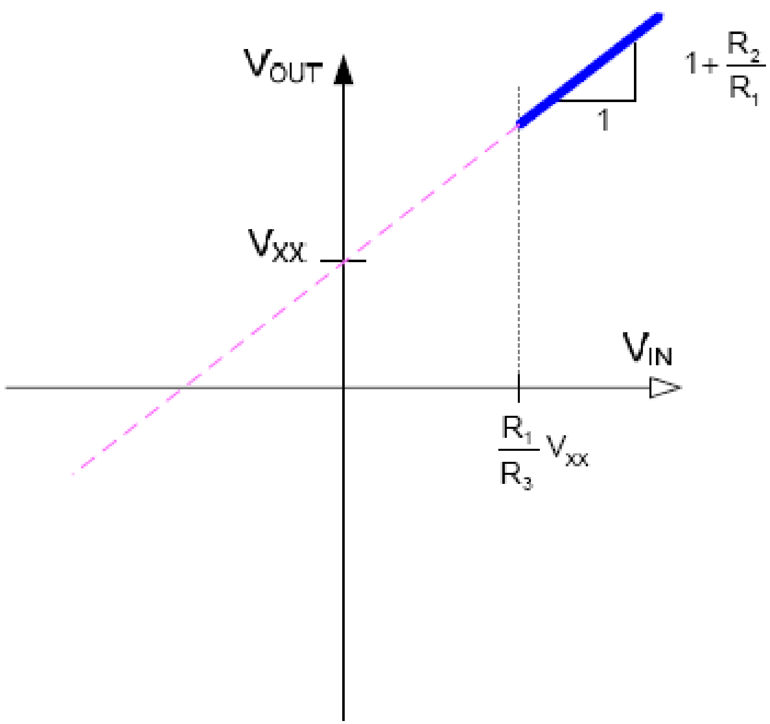
The first of these inequalities is valid provided $V_{XX}>0$ and substituting the expression for V_{OUT} into the second, we obtain after simplification

$$V_{IN} > \frac{R_1}{R_3}V_{XX}$$

Solution for Case 2 Continued

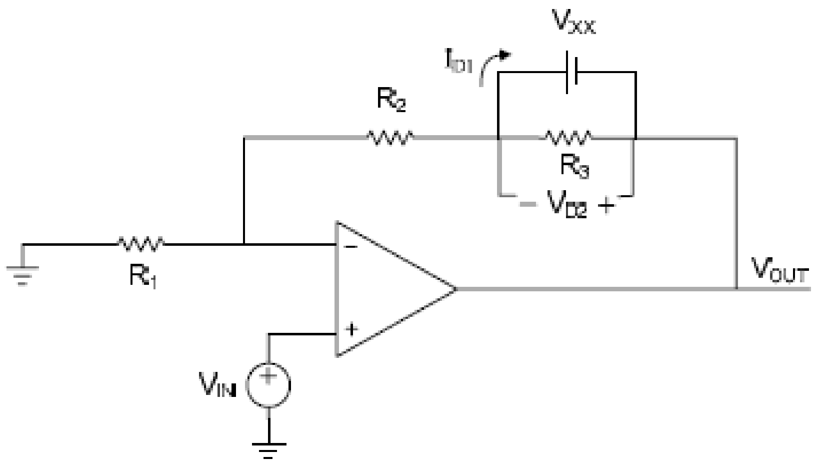
$V_{XX}>0$ $\qquad$ $V_{IN} > \frac{R_1}{R_3} V_{XX}$

Assuming $V_{XX}>0$, the region where Case 2 is valid is thus determined by the second inequality

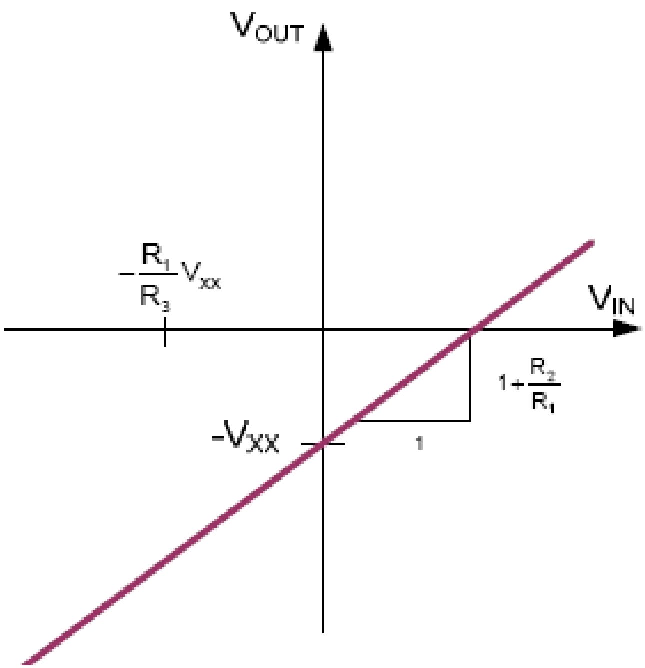


Case 3: NLD$_2$ is nonconducting ($I_{D2} = 0$)

NLD$_1$ is conducting ($V_{D1} = V_{XX}$)



$$V_{OUT} = \left(1 + \frac{R_2}{R_1}\right) V_{IN} - V_{XX}$$



Solution for Case 3 continued:

This solution is valid for $V_{D2} < 0$ and $I_{D1} > 0$

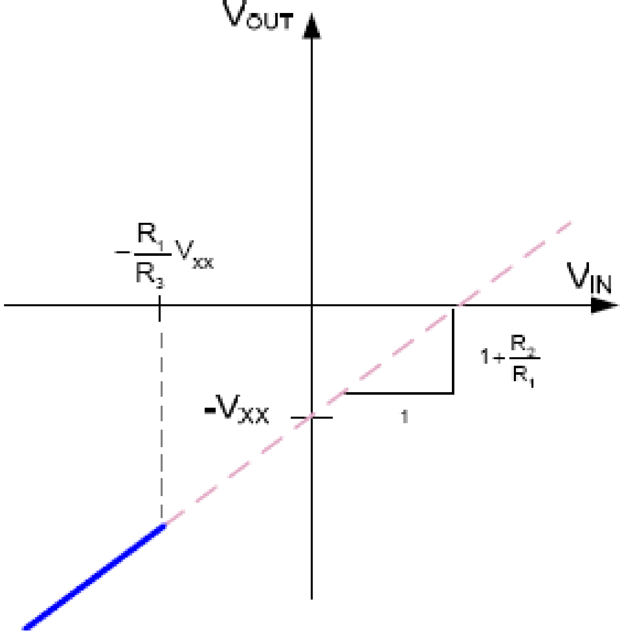
But: $V_{D2} = -V_{XX}$ and $I_{D1} = \frac{V_{IN} - V_{OUT} - V_{XX}}{R_2} - \frac{V_{XX}}{R_3}$

Substituting the validity conditions, we obtain

$-V_{XX} < 0$ and $\frac{V_{IN} - V_{OUT} - V_{XX}}{R_2} - \frac{V_{XX}}{R_3} > 0$

The first of these inequalities is valid provided $V_{XX} > 0$ and substituting the expression for V_{OUT} into the second, we obtain after simplification

$$V_{IN} < -\frac{R_1}{R_3} V_{XX}$$



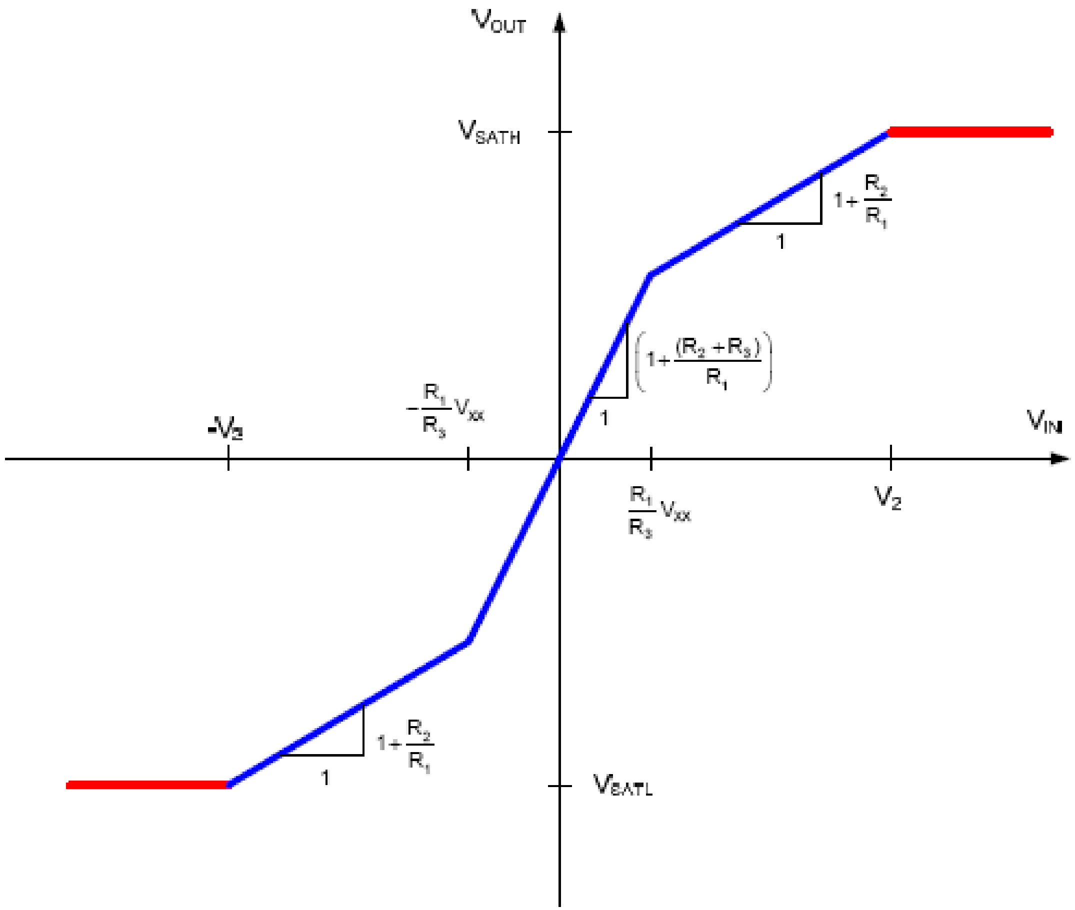
Case 4: NLD_1 and NLD_2 both conducting (this case never happens and need not be considered since we already have a solution for all inputs)

Thus, if we neglect the saturation of the op amp, we can write an expression for the output as

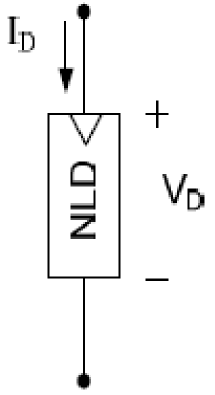
$$V_{OUT} = \begin{cases} \left(1+\dfrac{R_2}{R_1}\right)V_{IN} + V_{xx} & V_{IN} > \dfrac{R_1}{R_3}V_{XX} \quad \text{"2"} \\ \left(1+\dfrac{R_2+R_3}{R_1}\right)V_{IN} & -\dfrac{R_1}{R_3}V_{XX} < V_{IN} < \dfrac{R_1}{R_3}V_{XX} \quad \text{"1"} \\ \left(1+\dfrac{R_2}{R_1}\right)V_{IN} - V_{xx} & V_{IN} < -\dfrac{R_1}{R_3}V_{XX} \quad \text{"3"} \end{cases}$$

This is shown graphically, along with the saturation of the op amp, on the following slide

Overall Transfer Characteristics

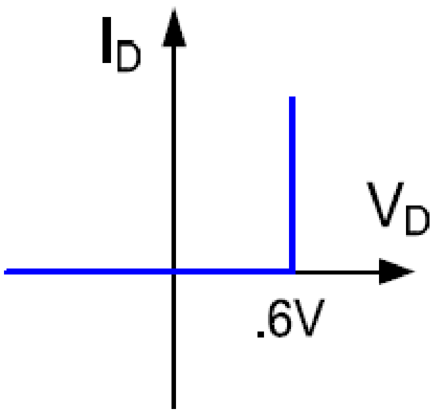


Overall Transfer Characteristics

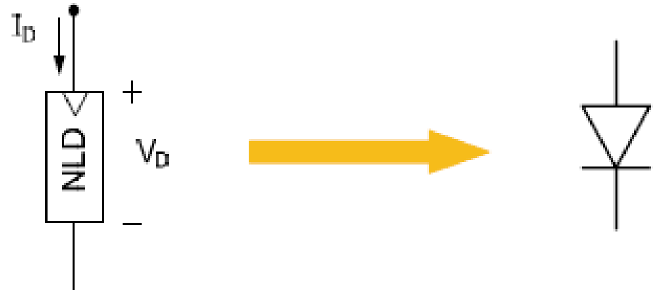


$V_D = V_{XX}$ for $I_D > 0$

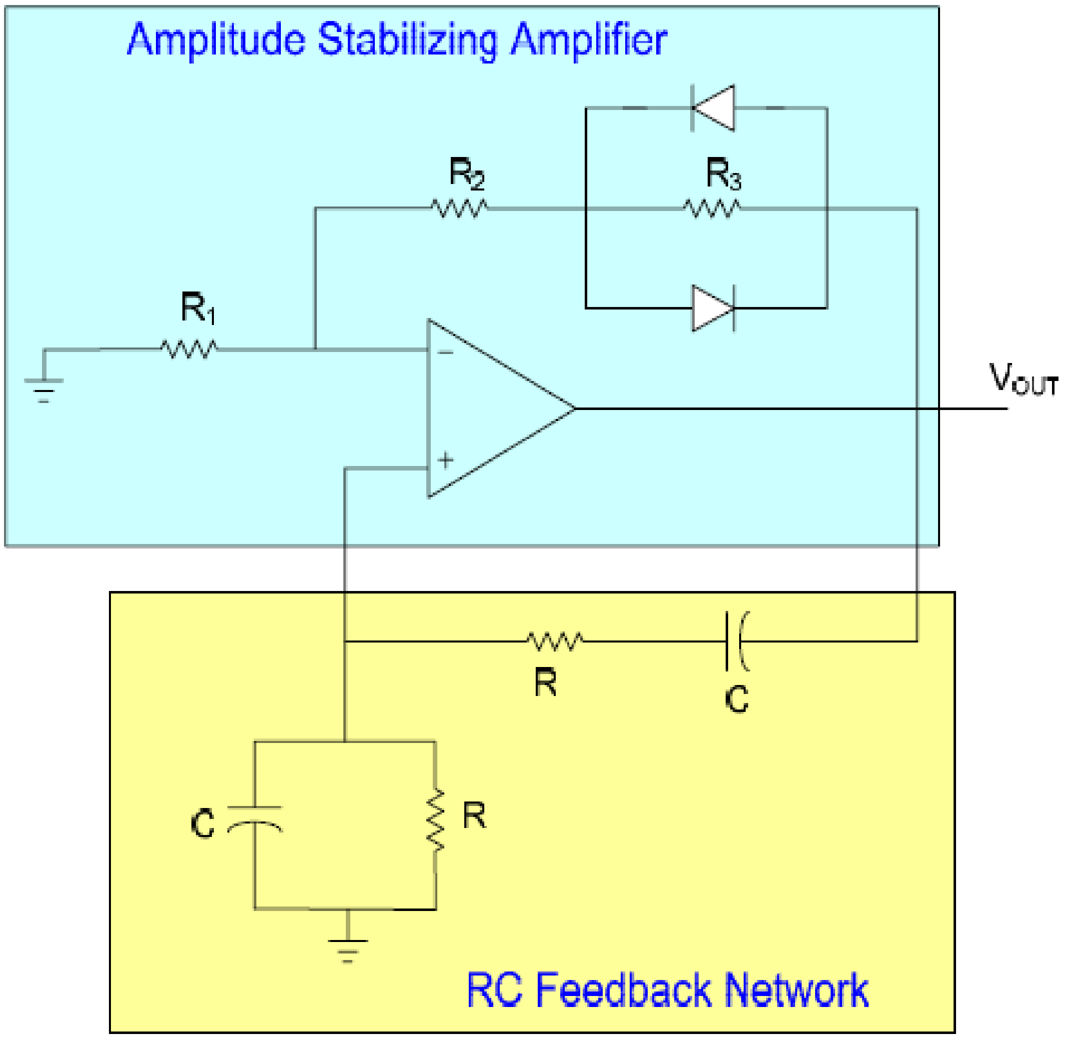
$I_D = 0$ for $V_D < V_{XX}$



If V_{XX}=0.6V, this represents a good approximation to the transfer characteristics of a silicon diode. We thus can replace the NLD with a diode and obtain the amplitude stabilized Wien-Bridge oscillator



Wein – Bridge Oscillator with Amplitude Stabilization



$$R_2 < 2R_1$$

$$R_2 + R_3 > 2R_1$$

$$\omega_{osc} \cong \frac{1}{RC}$$